REVISTA MEXICANA DE ECONOMÍA Y FINANZAS
Nueva Época
REMEF
(THE MEXICAN JOURNAL OF ECONOMICS AND FINANCE)

Revista Mexicana de Economía y Finanzas, Nueva Época

Volumen 21 Número 2, abril - junio 2026, pp. 1-22, e1384

DOI: https://doi.org/10.21919/remef.v21i1.1384

(Received: July 25, 2025; Accepted: December 15, 2025; Published: April 1, 2026)

imef fundación de investigación

Modelling the market dynamics of the exchange rate through Maxwell electrodynamic systems

Jennifer López-Chacon[1] - Universidad Michoacana de San Nicolás de Hidalgo, México
Martha Beatriz Flores-Romero[2] - Universidad Michoacana de San Nicolás de Hidalgo, México
Alfredo Raya[3] - Universidad Michoacana de San Nicolás de Hidalgo, México
José Álvarez-García[4] - Universidad de Extremadura: Cáceres, España

Abstract

We analyze the dynamics of the foreign exchange market (FOREX) within the framework of gauge theory, providing an interdisciplinary point of view that builds bridges among physics, mathematics and economics. We model the actual currency exchanges of euro-dollar-Mexican peso during the period of January to June 2023 through a one-dimensional electrodynamic system derived from Maxwell's equations. Findings of this study reveal that it is possible to describe out-of-equilibrium features and arbitrage conditions consistent with the FOREX dynamics. A natural outcome calls for the extension of the model including more currencies and real-time fluctuations to obtain a broader description of the market. The framework developed here provides a structural perspective to understand complex financial phenomena from theoretical physics arguments. An original feature of the present study is to translate the principles of electrodynamics into financial analysis, and thus, it offers a strong foundation to explore exchange rate dynamics and paves the way for further interdisciplinary research.

JEL Classification: C63, F31, G17, B4, D84.
Keywords: Econophysics, Gauge Theory, Currency Arbitrage, FOREX.

Modelación de la dinámica del mercado cambiario a través del sistema electrodinámico de Maxwell

Resumen

Analizamos la dinámica del mercado de divisas (FOREX) dentro del marco de la teoría gauge, proporcionando un punto de vista interdisciplinario que tiende puentes entre la física, las matemáticas y la economía. Modelamos las transacciones reales de cambio de euro-dólar-peso mexicano durante el período de enero a junio de 2023 mediante un sistema electrodinámico unidimensional derivado de las ecuaciones de Maxwell. Los hallazgos de este estudio revelan que es posible describir características fuera de equilibrio y condiciones de arbitraje consistentes con la dinámica del FOREX. Un resultado natural apunta a la extensión del modelo para incluir más divisas y fluctuaciones en tiempo real, a fin de obtener una descripción más amplia del mercado. El marco desarrollado aquí proporciona una perspectiva estructural para entender fenómenos financieros complejos a partir de argumentos de física teórica. Una característica original del presente estudio es traducir los principios de la electrodinámica al análisis financiero y, por lo tanto, ofrece una base sólida para explorar la dinámica de los tipos de cambio y allana el camino para futuras investigaciones interdisciplinarias.

Clasificación JEL: C63, F31, G17, B4, D84.
Palabras clave: Econofísica, Teoría de Gauge, Arbitraje de Divisas, FOREX.

[4] Corresponding Author. Email: pepealvarez@unex.es
*No funding source was received for the development of this research.

1. Introduction

The interaction among physics, mathematics, and economics has evolved over the centuries. The relationship of these fields dates to the emergence of classical mechanics and the formalisation of mathematical principles that began to influence economic ideas. The celebrated Newton's laws of motion and the law of universal gravitation formalised mathematically how bodies move around by the influence of forces. The principles of mechanics and the notion of equilibrium influenced the foundations of economics. Adam Smith's principle of the "invisible hand" (A. Smith, 1776) can be interpreted as an analogy to physical forces acting to guide an economic system towards equilibrium. Furthermore, Leon Walras (L. Walras, 1900) explicitly developed a mathematical description of market equilibrium in his formulation of general equilibrium theory in the late 19th century. The development of thermodynamics, particularly the laws of energy conservation and entropy in the XIX century provided further arguments to construct an analogy between energetic and economic systems. Nicholas Georgescu-Roegen (N. Georgescu-Roegen, 1971) argued that economic processes are subject to the second law of thermodynamics and introduced the concept of entropy into economics. This led to the development of ecological economics, emphasising that economic growth cannot occur without environmental constraints.

Furthermore, the development of quantum mechanics overcame the deterministic point of view that dominated physics and mathematics in the XX century. The probabilistic nature of quantum systems inspired new approaches in economic theory centred around complexity and uncertainty. Economists realized that markets are not always in equilibrium, and that economic behaviour may exhibit chaotic and unpredictable patterns, as in the subatomic realm. The field of complexity economics (see, for example, Ref. (Arthur, 2021)), which emerged from these ideas, considers the economy as a complex adaptive system.

During the mid-20th century, the advent of operations research during the Second World War demonstrated the practical use of mathematical techniques in economic planning and decision-making. John von Neumann (J. von Neumann et al., 2007) was among the first physicists and mathematicians who applied game theory and optimization methods to solve complex problems of logistics and resource allocation inspired in the military industry. These ideas were naturally extended to economics, particularly in industrial planning and scientific management.

More recently, physics-inspired models such as network theory (see Ref. (M. O. Jackson, 2009)) have increased attention in economics, where it is employed to model the interconnectedness of economic systems, including financial markets and supply chains. This interdisciplinary framework puts forward the systemic risks that arise from economic interconnection and has been crucial in understanding phenomena such as the 2008 financial crisis.

A novel connection between gauge theory within the context of quantum field theory and economics, particularly in financial markets, was developed by K. Ilinsky, (K. Ilinski, 2001) to model out-of-equilibrium dynamics in economics. Gauge theory is a core component of the Standard Model of Particle Physics from local symmetries, where different components behave independently but remain connected through gauge fields. Ilinsky extended this concept to financial markets, which are systems far from equilibrium that fluctuate and display nonlinear dynamics. His model treats asset prices as fields influenced by a "gauge connection" which, analogously in particle physics, gives rise to external fields such as in quantum electrodynamics. This framework offers a new perspective for pricing financial derivatives under out-of-equilibrium conditions, capturing the stochastic nature of markets. As opposed to traditional models based on the Black-Scholes equation, the gauge theoretical model reflects the dynamical, fluctuating, and discontinuous behavior of real markets, providing deeper insights into pricing and market crashes. Gauge theory highlights the role of arbitrage in finance, which drives markets towards equilibrium but rarely keeps them there for long. This interdisciplinary approach has inspired further research at the intersection of physics, economics, and mathematics (see, for example, Ref. (B. Baaquie, 2007)). A recent account of the historical development of gauge theories in financial markets is presented in (A. Raya et al., 2025).

An idealized model inspired by gauge theory to describe financial markets was introduced in Ref. (J. Schwichtenberg, 2010) in which the financial market is considered as an electrodynamical system where traders are treated as particles and their interactions are governed by external forces, represented as gauge fields. The author uses this framework to describe how local interactions among traders, influenced by such external factors, can lead to out-of-equilibrium price dynamics. By borrowing mathematical tools from gauge theory, the model provides insights into the stochastic and often chaotic nature of financial markets, capturing phenomena such as price fluctuations and the propagation of systemic risk.

In this article, we follow these ideas and describe FOREX dynamics in terms of a one-dimensional electrodynamic system, featuring simplified feedback between traders and influential factors that drive out-of-equilibrium price dynamics. For this purpose, the remainder of the article is organised as follows. The next section reviews the general features of Schwichtenberg's model and presents our adaptation to FOREX, i.e., the global decentralised market where currencies are traded. Among many other features, FOREX facilitates the exchange of one currency for another, enabling international trade, investment, and financial operations 24 hours a day. It is the largest and most liquid financial market in the world, with daily trading volumes exceeding trillions of dollars, with participants that include banks, financial institutions, corporations, governments, and individual traders. Currency exchange rates in this market are influenced by various factors, including economic indicators, geopolitical events, interest rates, and market speculation. Section two is dedicated to documenting previous research where gauge theory has been used to model the foreign exchange market. Section three outlines the general characteristics of market dynamics, while section four describes the specific dynamics of the FOREX market. In section five, we present the physical interpretation of the currency market and outline the methodology used for our research purposes. This section also presents the results obtained from analyzing exchanges between US dollars and euros during the period from 1 January to 12 June 2023. The discussion of our findings and the conclusions drawn from them are presented at the end, in section seven.

2. Literature Review

This section presents, in chronological order, the most relevant articles and books related to the application of gauge theory to the field of finance, particularly in the foreign exchange market (FOREX). The approach to the foreign exchange market from the perspective of symmetries and gauge theory was first introduced in 1996 in Malaney's doctoral thesis, "The Index Number Problem: A Differential Geometric Approach." In this work, the author argues that symmetries—and in particular, gauge theory—are applicable to the field of economics, drawing partially on a joint paper with Weinstein (Malaney, 1997).

Shortly thereafter, in 1997, Ilinski and Kalinin focused their efforts on applying gauge theory to arbitrage in derivative pricing. They analyzed the Black-Scholes model, deriving the Black-Scholes equation from gauge theory arguments. Furthermore, they developed a correction to the said equation that accounts for the impact of virtual arbitrage and speculator reactions. Although the results in that work were drawn analytically, the authors emphasized that future developments would require numerical methods to solve more complex calculations (Ilinski & Kalinin, 1997).

Didier Sornette (Sornette, 1998) analyzed the proposal of Ilinski and Kalinin, arguing that neither the lognormal distribution nor the Black-Scholes equations provide convincing evidence supporting the validity or relevance of the theory, suggesting that the market uncertainty is the source of virtual arbitrage opportunities. However, in a complete market, random variables would be led by a random walk process, implying the absence of arbitrage opportunities.

Application of gauge theory in economics were pedagogically discussed in (Ilinski, 1998), exploring, for instance, modeling of real price processes for the S&P500 index, as well as the analysis of derivative pricing and portfolio theory. An algorithm to develop a pricing model for a specific financial instrument within the gauge symmetry framework was further elaborated in that reference.

An analogy between a simplified model for the foreign exchange market and a lattice gauge theory (Young, 1999) suggests that exchange rates could be interpreted as exponentials of gauge potentials defined over spatial loops, while interest rates are linked to gauge potentials over temporal loops. Arbitrage opportunities arise from non-zero values in the gauge-invariant field tensor or from curvature over closed loops. In spite of the fact that this framework is basic and does not fully reflect the real behavior of a financial market, it offers a novel perspective for analyzing the foreign exchange market and lays the groundwork for the development of more complex models.

More recently, Ilinski (Ilinski, 2000) proposed that financial markets possess an intrinsic geometric structure of fiber bundles, which allows for the formulation of local gauge symmetry to rescale asset units using purely geometric concepts. By incorporating this perspective into the construction of financial models, a methodology based on physical principles can be straightforwardly introduced, significantly enriching the analysis within financial economics.

In a series of articles, Kholodnyi (Kholodnyi, 2002; Kholodnyi & Trading, 2003) offers a new perspective on gauge symmetry grounded in agents' beliefs and preferences, along with significant advancements in the valuation of European options in general markets where price dynamics do not necessarily follow Markov processes.

In the article "Time and Symmetry in Models of Economic Markets", Smolin addressed various economic issues from an alternative perspective, aiming to analyze market modeling. He described the Arrow-Debreu general equilibrium model, focusing on the treatment of time and contingency, and concluded that it is necessary to move toward a dynamic, out-of-equilibrium theory for economic markets, using the Debreu model as a base. He proposed calling this approach "statistical economics" and suggested studying the economy through the lens of gauge theory, in line with the original proposals of Malaney and Weinstein (Smolin, 2009). Also in 2009, Samuel E. Vázquez and Simone Farinelli published the article "Gauge Invariance, Geometry and Arbitrage", in which they proposed a general arbitrage measure applicable to any market model based on Itô processes. They demonstrated that this measure is invariant under changes in the numéraire and equivalent probability measures, interpreting it geometrically as a gauge connection. Moreover, the authors presented an algorithm to obtain the market curvature from financial data, identifying non-zero curvature fluctuations in high-frequency data related to stock and futures indices (Vázquez & Farinelli, 2009).

Further studies on the topic include (Morisawa, 2009; Sokolov et al. 2010), where triangular arbitrage and rapid monetary flow dynamics are, respectively, explored. Moreover, recent research articles include (Zhou and Xiao, 2010; Maldacena 2015; Farinelli 2015) which have expanded the applications of gauge theory in derivative pricing and financial market dynamics. Finally, the book in Ref. (J. Schwichtenberg, 2020) and recent thesis projects at the Universidad Michoacana de San Nicolás de Hidalgo (Ramos-Llanos, 2022; Servín , 2023) emphasize the use of simplified models to understand arbitrage and the dynamics of the foreign exchange market, showing how gauge theory continues to serve as a valuable framework for financial analysis.

3. Maxwell's Equations and the Financial Market

In this section, we present a summary of the classical theory of electrodynamics, which is summarized in the well-known Maxwell's equations

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}, \tag{1}$$

$$\nabla \cdot \vec{B} = 0, \tag{2}$$

$$\nabla \times \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}, \tag{3}$$

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0, \tag{4}$$

and describe how the electric field $\vec{E}$ and the magnetic field $\vec{B}$ (each having three spatial components that may depend on time) are generated by their sources, ρ and $\vec{J}$, which represent the charge and current densities, respectively, how they interact with these sources, and even how they behave in the absence of them. Here, the constants μ_0 and ϵ_0 are the permeability and permittivity of vacuum, respectively. The sources are connected through the continuity equation.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0, \tag{5}$$

which, from a physical point of view, represents the conservation of electric charge in any electromagnetic process. The full set of Maxwell's equations is rarely solved directly for the electric and magnetic fields. Only in highly symmetric situations is it possible to solve these equations for the six components of the $\vec{E}$ and $\vec{B}$ fields. However, we can observe that these fields can be expressed in terms of a scalar potential A_0 and a vector potential $\vec{A}$ as:

$$B = \nabla \times \vec{A}, \quad \vec{E} = -\nabla A_0 - \frac{\partial \vec{A}}{\partial t}. \tag{6}$$

Then, the set of Maxwell's equations is reduced to four equations for the four components of A_0 and $\vec{A}$. However, the choice of these auxiliary potentials is not unique. There exists a residual gauge freedom in which we can choose:

$$\vec{A} \rightarrow \vec{A}' = A + \nabla \lambda, \quad A_0 \rightarrow A'_0 = A_0 - \frac{\partial \lambda}{\partial t}, \tag{7}$$

where λ is a scalar function, called the gauge function. A suitable choice of λ makes the equations easier to solve. However, under these transformations, neither of the fields $\vec{E}$ and $\vec{B}$ changes, and therefore, the transformations in Equation (7) are said to be a symmetry of Maxwell's equations—gauge symmetry. A_0 and $\vec{A}$ are known as the gauge potentials or gauge connections. Gauge symmetry is a profound concept that transcends physics, offering a framework for understanding invariance, conservation, and redundancy.

In electrodynamics, we consider the sources ρ and $\vec{J}$ that are localized in space at a given moment. However, for the financial market model, we require these functions to exhibit a stochastic nature, as we will explain shortly when addressing the FOREX market. For simplicity, we require that the charge density be constrained to satisfy the gauged diffusion equation.

$$\left(\frac{\partial}{\partial t} - A_0\right)\rho(x,t) = D\left(-i\nabla - \vec{A}\right)^2\rho(x,t), \tag{8}$$

where, as usual, the partial derivatives are replaced by covariant derivatives through the incorporation of A_0 and A_0, the gauge potentials.

3.1. One-Dimensional Model

In one spatial dimension, a magnetic field cannot be defined, and therefore $\vec{B} = 0$. This means that only three of Maxwell's four equations remain, and they take the following form:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}, \tag{9}$$

$$\epsilon_0 \frac{\partial \vec{E}}{\partial t} = \vec{J}, \tag{10}$$

$$\nabla \times \vec{E} = 0. \tag{11}$$

From the last expression (11), we can choose to define the electric field instantaneously in terms of the gradient of a scalar function. In this way, by opting to work in the Coulomb gauge, we consider the following.

$$\vec{E} = -\nabla A_0, \tag{12}$$

so that equation (9) takes the form of a Poisson equation.

$$\nabla^2 A_0 = \frac{\rho}{\epsilon_0}, \tag{13}$$

which we can formally solve as.

$$A_0(x,t) = \frac{1}{\epsilon_0}\int d^3x' G(x-x')\rho(x',t). \tag{14}$$

where $G(x-x')$ is the Green's function of the one-dimensional Laplacian, that is, the function that solves the Poisson equation with a point source.

$$\nabla^2 G(x-x') = 4\pi\delta(x-x') \tag{15}$$

Substituting (14) into Equation (8), we derive the equation of motion for the charge density.

$$\frac{\partial \rho(x,t)}{\partial t} = D\frac{\partial^2}{\partial x^2}\rho(x,t) + \left[\frac{1}{\epsilon_0}\int d^3x' G(x-x')\rho(x',t)\right]\rho(x,t) \tag{16}$$

This nonlinear integro-differential equation is a challenging problem to solve, both analytically and numerically. The integral in brackets on the right-hand side drives the nonlinear and stochastic dynamics of the financial market.

Instead of attempting to solve Equation (16) directly, we introduce a quenched approximation in which we solve the simplified equation.

$$\frac{\partial\rho(x,t)}{\partial t} = D\frac{\partial^2}{\partial x^2}\rho(x,t) + V(x)\rho(x,t). \quad (17)$$

In this expression, the function $V(x)$ encodes a phenomenological model of the dynamics. From our point of view, we infer it from the behaviour of the financial market. Below, we present the main elements needed

4. FOREX

The foreign exchange market, better known as FOREX, traces its origins back to ancient times, when early civilisations began to trade goods and services using coins and commodities of different regions. Gold and silver were among the first forms of currency, valued for their rarity and stability. As trade routes expanded, the need to establish consistent systems of exchange became more evident, which gradually led to the creation of formal monetary mechanisms. The contemporary FOREX market, however, took shape after the 1971 collapse of the Bretton Woods agreement, when the United States ended the convertibility of the dollar into gold. From that moment, currencies were allowed to fluctuate freely according to market forces of supply and demand, marking the start of the 7tabilize7ion and flexible exchange system that underpins today's global economy (Mishkin & Eakins, 2016).

Two major forces have driven the evolution of this market: 7tabilize7ion and technological innovation. During the 1980s, the implementation of electronic trading platforms transformed how financial institutions conducted currency operations, increasing both the speed and efficiency of transactions. Initially, these activities were almost exclusively in the hands of central banks, investment funds, and multinational corporations. The advent of the internet in the 1990s changed that scenario drastically, allowing individual investors to participate through online brokers. This development opened the market to retail traders, multiplying its volume and accessibility worldwide (Fant, 1999).

Today, FOREX operates continuously, 24 hours a day from Monday to Friday, connecting financial hubs such as Tokyo, London, and New York in overlapping sessions. Unlike stock exchanges that operate in fixed locations, FOREX functions as an over-the-counter network, with transactions occurring directly between participants. With a daily turnover exceeding six trillion dollars, it is considered the largest and most liquid financial market in existence, reflecting its central role in the world economy (Canton, 2021). Currency trading is based on pairs, where one currency is exchanged for another—for example, EUR/USD or GBP/JPY. The value of each pair fluctuates according to a complex set of economic and political variables, including inflation rates, monetary policies, trade balances, and geopolitical events. These interrelated factors make the market highly dynamic and sensitive to global developments, requiring constant monitoring by institutions and investors alike (Engel & West, 2005).

The variety of agents involved adds to the market's depth. Central banks intervene to 7tabilize their currencies; corporations use FOREX to hedge against risks arising from international transactions; and private traders speculate on short-term price movements, taking advantage of small fluctuations for profit (Lyons et al., 2001). This diversity ensures strong liquidity and generally low transaction costs. Empirical evidence from crisis episodes also shows that volatility spillovers intensify across markets and that investors actively seek hedging and diversification strategies using alternative assets, reinforcing the relevance of risk-hedging behavior in turbulent periods (Vairasigamani & Amilan, 2025).

Nevertheless, despite its stability in terms of volume, FOREX remains vulnerable to abrupt episodes of volatility triggered by unforeseen events—political crises, wars, or sudden policy shifts. Such occurrences can cause sharp variations in exchange rates, turning the market into a space of both opportunity and uncertainty. Statements by central banks or unexpected geopolitical developments often lead to immediate and significant changes in investor sentiment (Chaboud et al., 2023). Recent event-study evidence shows that geopolitical conflicts can trigger statistically significant reactions in capital markets and macroeconomic indicators across countries, reinforcing the role of geopolitical shocks as a source of abrupt shifts in expectations and volatility in international financial markets (Rochimah & Yuliana, 2025).

Beyond speculation, FOREX fulfils an essential macroeconomic role: it facilitates international trade and capital flows by enabling the conversion of currencies efficiently. It also provides signals that reflect economic conditions and serve as references for monetary and fiscal decision-making (Krugman, 2009). Looking forward, technological innovation continues to redefine the structure and operation of this market. Tools such as algorithmic trading, artificial intelligence, and blockchain are introducing new levels of automation, precision, and transparency. These transformations are not merely technological; they reshape the very logic of global finance, ensuring that FOREX remains one of the central pillars of economic interconnection in the twenty-first century (Harris, 2002).

5. Physical Interpretation of the Financial Market

One of the most important advances in describing the fundamental constituents of the universe and their interactions is to recognize that in addition to space-time, there also exist internal spaces, which mathematically permit the description of objects that acquire a physical reality that can be measured. The combination of space-time with internal spaces forms the foundation of modern physics. This structure is not static, but rather has a dynamic nature (Schwichtenberg, 2019).

To understand the impact of internal spaces, we require to introduce the notion of transformations and symmetries. A group is a mathematical structure used to this end. It consists of a set of objects or elements and a defined operation or combination among the elements of the group that satisfies certain properties, such as closure, associativity, the existence of an identity element, and the existence of an inverse for every element in the group. In the context of symmetries, a group is formed by the set of all transformations that leave a specific object invariant. These transformations can be rotations, reflections, and translations that preserve the shape or configuration of the object under study. They can also be abstract mathematical transformations. The operation, commonly called "composition", combines two transformations to produce another within the group (Stillwell & Stillwell, 1989).

Groups can be discrete, as in the case of the rotations that leave a cube invariant, or continuous, as in the case of the rotations of a sphere. Discrete groups have a finite number of elements, while continuous groups have an infinite number. Fundamental interactions are described through transformations carried out in the internal space of physical systems of interest, such as particles. Some transformations do not depend on the space-time coordinates in which the particles move, and are known as global transformations. Others, known as local transformations, do depend on these coordinates. Some of these transformations allow us to describe the dynamics of elementary particles in terms of conserved charges and currents (Peskin, 2018).

Gauge symmetry is a fundamental principle in physics that describes the invariance of physical laws under local transformations of the fundamental fields of a system. These transformations depend on space-time coordinates and require the introduction of gauge fields to ensure invariance. Electrodynamics is an iconic example of a "gauge theory". The fact that gauge potentials can be chosen without compromising the actual electric and magnetic fields is the cornerstone of modern theories of fundamental interactions.

This framework provides a natural way to understand the interactions between electromagnetic fields, electric charges, and currents. This approach is particularly useful for generalising such ideas to other interactions and for describing phenomena in the realm of quantum theory (Maldacena, 2015). The analysis of the foreign exchange market can be carried out establishing analogies between physical interactions and economic phenomena under the grounds of gauge theory. Indeed, within this framework, exchange rates can be interpreted as gauge fields, whereas currencies represent states. The former act as connections that describe the relationship between them and the latter in a closed system. This approach offers a systematic way of modelling the consistency in exchange rate relationships (Schwichtenberg, 2019).

In this framework, arbitrage is associated with curvature in the internal space of a physical system. Non-zero curvature indicates the existence of arbitrage opportunities, while a system with zero curvature reflects a market in equilibrium. Thus, arbitrage becomes a direct consequence of the internal dynamics of the market. Furthermore, this point of view distinguishes between global and local transformations. Global transformations, such as changes in international interest rates, affect all currencies uniformly, while local transformations reflect specific events influencing a particular currency, such as national economic policies (Vázquez & Farinelli, 2009).

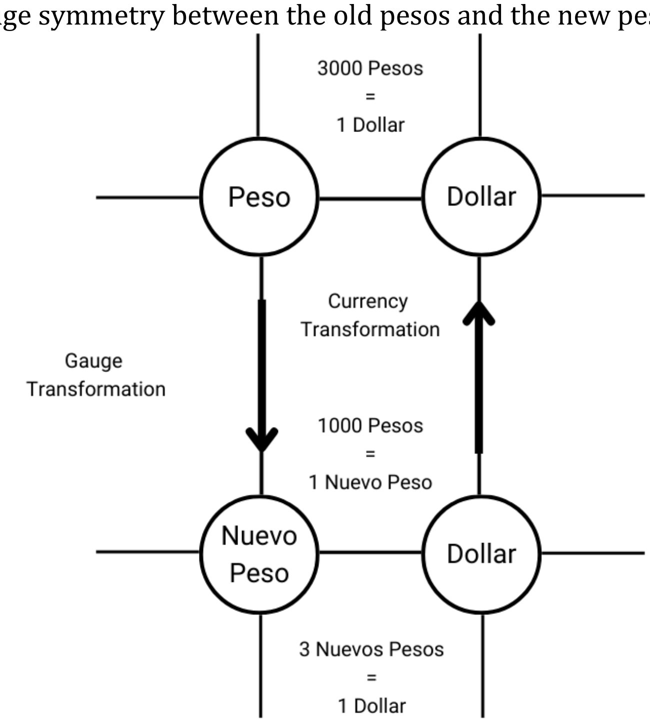
From a mathematical point of view, Maxwell's equations in one spatial dimension can be adapted to describe the time evolution of exchange rates and their response to external factors, capturing the nonlinear dynamics, extreme volatility, and currency crises, for example. The gauge theory-inspired approach we are proposing provides robust numerical tools to identify arbitrage opportunities, assess risks, and explore complex interactions in the foreign exchange market (Ilinski, 2001). Gauge theory provides a powerful framework for analysing the complexities of the foreign exchange market by establishing parallels between financial and physical variables such as fields, connections, and curvature. It offers a structured approach to model exchange rate dynamics, including nonlinear behaviours, volatility, and crises. Moreover, arbitrage opportunities are interpreted as non-zero curvatures in the internal gauge space, whereas equilibrium corresponds to zero curvature. This framework also incorporates the effects of external factors, such as monetary policies and global interest rates, enabling advanced simulations and predictive models to assess risks, explore interactions, and identify profitable opportunities in financial markets (Ilinski, 2001).

5.1. Depreciation of a Currency

Let us begin by considering the financial market as a physical system where banks from different countries are arranged in a grid, and each country has its own currency. These banks are interconnected, but remain completely free to set whichever exchange rates they wish, without charging fees. Money can only be transferred between banks. Now, to identify a gauge symmetry in this context, let us consider a country which, due to high inflation, decides to remove zeros from its currency—a common measure in situations of severe inflation, i.e., the local government decides to change its monetary units (Maldacena, 2015).

This actually occurred in Mexico during the 1980s, when inflation significantly increased prices and high-denomination banknotes were issued, causing uncertainty about the actual reserves of silver and gold. To address this situation, on 22 June 1992, the introduction of a new currency, the "Nuevo Peso," was decreed, removing three zeros from the previous peso. Thus, 1,000 pesos from 1980 became equivalent to 1 Nuevo Peso (N$) as of January 1993. For example, if the previous exchange rate was 3,000 pesos per dollar, with the Nuevo Peso it was adjusted to 3 Nuevos Pesos per dollar. This change did not alter wealth or economic opportunities, either in Mexico or abroad. This reflects a local gauge symmetry in the international market, since everything remained equivalent, modified only for convenience. This example is illustrated graphically in Figure 1.

Figure 1: Gauge symmetry between the old pesos and the new pesos of Mexico in 1993.



Source: Own elaboration, based on Maldacena (Maldacena, 2015).

This local gauge symmetry can be observed in action in some Mexican coins depicted in Figure 2. It is, in fact, a gauge symmetry because it necessarily implies price invariance for services and/or goods in the units used to measure or "calibrate" the value of different quantities. We emphasise that it is a local symmetry, since each country is free to make this adjustment independently, without being influenced by the decisions of neighbouring countries. Some countries resort to this type of change more frequently than others (Maldacena, 2015).

Figure 2. Equivalences of currencies from the 1980s with the new pesos.

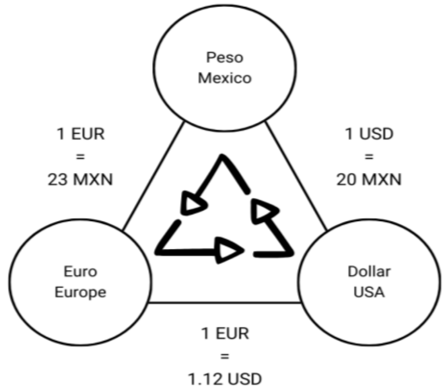
Source: https://www.elfinanciero.com.mx/economia/te-acuerdas-cuando-eras-chiquito-y\-el-dolar-valia-3-pesos/.

5.2 Currency Arbitrage from Gauge Theory

A speculator is an agent who participates in financial markets by buying and selling assets, such as securities, bonds, or currencies, with the aim of making profits from price fluctuations in the market in which they operate. In the Forex market, as described, a speculator seeks to capitalise on variations between currencies to generate gains.

Consider three countries: the United States, France, and Mexico, with their respective currencies, namely dollars, euros, and pesos. Suppose the countries are connected as illustrated in Figure 3, and the exchange rates are as follows: 1.12 dollars = 1 euro, 1 dollar = 20 pesos, and 1 euro = 20 pesos. In this situation, it is possible to make a profit by completing a circuit: starting in Mexico with 20 pesos, one can buy 1 euro in France, then exchange the euro for 1.12 dollars in the United States, and finally return to Mexico to convert the dollars back into pesos, obtaining 22.4 pesos. This results in an outcome of 12% profit of the initial amount which does not depend on the monetary units used (Maldacena, 2015).

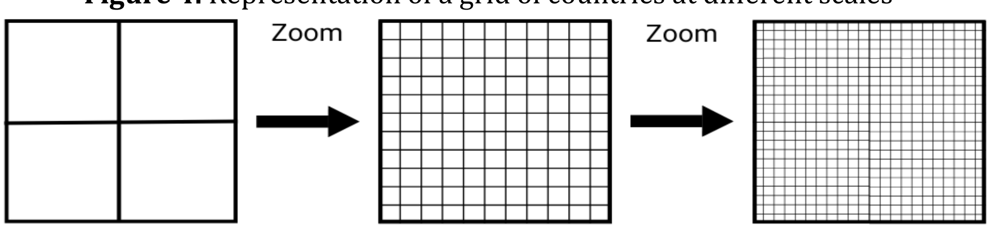
Figure 3. Illustration of three countries and their respective currencies[5]



Source: Own elaboration, based on Maldacena (Maldacena, 2015).

This example might lead us to think that banks do not set exchange rates correctly, but this is not the case. Although banks fix a price for currencies, this price varies according to supply and demand dynamics. Therefore, profit opportunities are not always present nor constant. Speculators seek to maximise their gains and will choose the routes that provide the highest profits. In the case described, speculators would follow a circuit moving between Mexico, France, and the United States, returning to Mexico, as shown by the thick line in Figure 3. From a physics perspective, countries are interpreted as points in space, while exchange rates represent configurations of connections (or electromagnetic potentials) in that space. Let us suppose that this example occurs at extremely small scales, beyond what we can actually measure. Observing a physical system from a distance, it would appear continuous, as represented in Figure 4. Similarly, an electron moving in a vacuum easily travels from one point in space-time to another. At a microscopic level, it would be as if the electron were constantly switching between countries and accumulating profits in the process. In physics, it is not known for certain whether an underlying discrete structure exists similar to the countries described. However, when performing calculations in gauge theories, it is often assumed that a discrete structure exists, which is then extrapolated to the continuous limit, where regions are infinitesimally close to each other (Maldacena, 2015).

Figure 4. Representation of a grid of countries at different scales[6]



Source: Own elaboration, based on Maldacena (Maldacena, 2015).

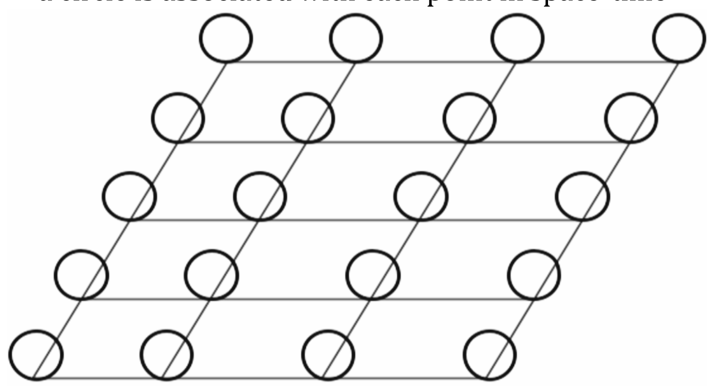
[5] The bridges indicate the established exchange rates among them. Following the circuit represented by the thick line, it is possible to make a profit, resulting in a factor of 1.12, equivalent to a 12% increase.

6 A perspective of this grid from a considerable distance is shown. When viewed from far enough away, the grid appears to be a continuous system.

As we have emphasised throughout this manuscript, electrodynamics possesses a gauge symmetry. This symmetry allows transformations in space-time of the electromagnetic connections or potentials, which vary at each point in space-time without altering the values of the electromagnetic fields. From a mathematical perspective, this symmetry is analogous to rotations of a circle around an axis passing through its centre. One way to visualise this concept is to imagine that at each point in space-time there exists an additional circle, an extra internal dimension (see Figure 5). In this case, each "country" located at a point in space-time defines a frame of reference to measure angles on that circle, choosing a point as "angle zero" and describing the relative positions of other points based on this. This is similar to the choice of a currency in the example of the three countries.

In physics, there is no evidence that this additional circle is real or that such an extra dimension exists. It is only recognised that the symmetry is consistent with observations, without the need to assume the existence of such a dimension. The relevant quantities are the electromagnetic potentials, which describe how the position of a particle on the additional circle changes as it moves from one point in space-time to a neighbouring one (Maldacena, 2015).

Figure 5. The electromagnetic interaction exhibits symmetries equivalent to a configuration where a circle is associated with each point in space-time[7]



Source: Own elaboration, based on Maldacena (Maldacena, 2015).

In electrodynamics, the electric and magnetic fields obey Maxwell's equations. That is, Maxwell's equations are the dynamic equations of these fields, their equations of motion. In economic terms, this can be interpreted as a constraint on exchange rates. Intuitively, in an economic model, this requirement can be explained as follows: imagine a scenario with generic exchange rates where speculators transfer money between different currencies. If we consider a specific "bridge" connecting two particular currencies, there will be speculators crossing it in both directions. However, if the flow of speculators is greater in one direction than the other, the bank at that bridge could run out of one of the currencies.

For example, if a bank at a bridge connecting pesos and dollars notices that more speculators want to buy dollars than pesos, it could eventually exhaust its dollar reserves. In this case, the bank would adjust the exchange rate to reduce the demand for dollars and balance the flow. Assuming that the number of speculators following a circuit is proportional to the profits obtained along that route, the condition for banks to maintain equilibrium — that is, for the net flow of money across each bridge to be zero — turns out to be equivalent to Maxwell's equations (Maldacena, 2015).

[7] In this representation, the points in space-time are at the intersections of the black lines. The circle can be interpreted as an additional internal dimension.

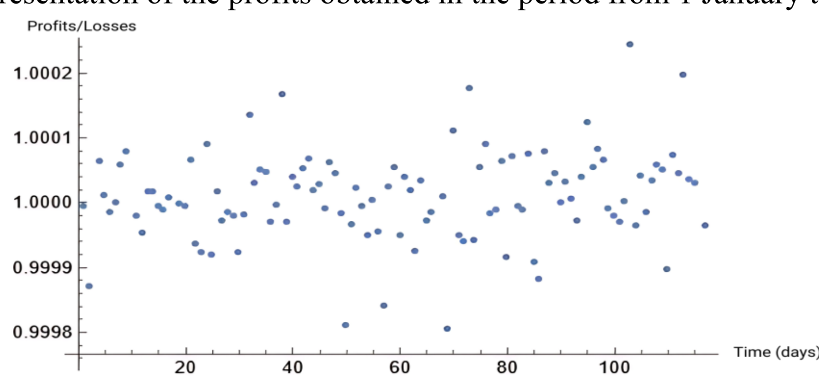
6. Methodology

We have already introduced a perspective on financial markets by drawing an analogy with the gauge theory of electrodynamics, presenting a novel framework to conceptualise and model the dynamics within these markets. This approach not only establishes a connection between the mathematical structures of physics and finance but also provides means to model the complex interactions of financial systems as interactions in physical systems. In particular, by using concepts from electrodynamics, we can describe the movement of assets, market forces, and potential imbalances in a way that resembles the interaction of charges and fields in physical systems. To ensure clarity and accessibility, we have synthesised this perspective into a set of Maxwell's equations that serve as a basis to explore how market forces evolve over time and respond to external influences, analogously to interactions in electrodynamical systems governed by field equations. The simplified model reduces the complexity of real-world financial markets but retains sufficient detail to capture essential features such as market volatility, asset price fluctuations, and interactions among market participants. The most general form of the equations that govern the market dynamics is

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \quad Gauss' \ Law$$
$$\nabla \cdot B = 0 \quad Magnetic \ Gauss' \ Law$$
$$\nabla \times E = -\frac{\partial B}{\partial t} \quad Faraday's \ Law$$
$$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \quad Ampere's \ Law$$
$$\nabla \cdot J = \frac{\partial \rho}{\partial t} \quad Continuity \ Equation$$
$$D_t \rho = -D D_x^2 \quad Gauged \ Diffusion \ Equation$$

We decided to work with the euro and dollar currencies in the following exercise. We created a database that allowed us to obtain the exchange rates between these currencies. The data corresponding to the period from 1 January to 12 June 2023 were taken from the Investing website. This database was created using Google Sheets and later saved in CSV format. The data were processed in Mathematica. Our main objective is to analyse the profits obtained considering the opening price. From the recorded data, we obtained the graph shown in Figure 6.

Figure 6. Representation of the profits obtained in the period from 1 January to 12 June 2023.



Source: Own elaboration.

7. Results

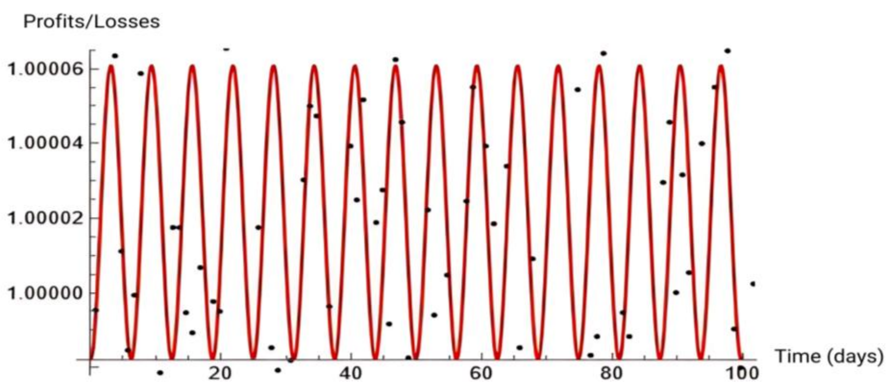
In Figure 7, we can observe that profits are obtained when the result is greater than one, there are neither profits nor losses if the result is exactly one, and losses occur if the result is less than one. Once these changes were calculated, we proceeded to work with these results using Maxwell's equations. First, we approximate the financial market as a one-dimensional problem, so that the equations become ($\vec{B} = 0, \nabla \times \vec{E} = 0$):

$$\frac{\partial \vec{E}}{\partial x} = \frac{\rho}{\epsilon_0}, \frac{\partial \vec{E}}{\partial t} = \mu_0 J, \frac{\partial J}{\partial x} = \frac{\partial \rho}{\partial t}, D_t \rho = -D\frac{\partial^2 \rho}{\partial x^2}.$$

Let us consider a quenched approximation, where the rules of the game are set from the beginning and do not change. This allows us to answer two types of questions:

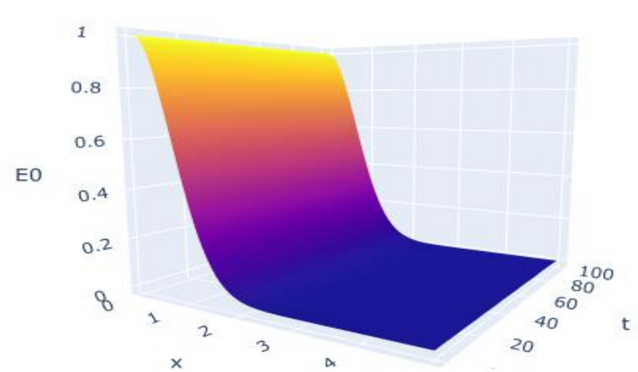
If we want a specific behavior in the gains represented by $\vec{E}$, what should be the market dynamics encoded by ρ to make this possible? If we know the market dynamics, parameterized by ρ, what are the expected gains? With these ideas in mind, we model the foreign exchange market. We first propose a profit profile that is spatially localized at time t = 0 and provides a cosine-like market evolution for t > 0, either with an upward or downward trend, as follows:

Figure 7. Cosine fit to the obtained gains, with the function. $Ae^{-x^2}(1 - B\ Cos(c - t))$.



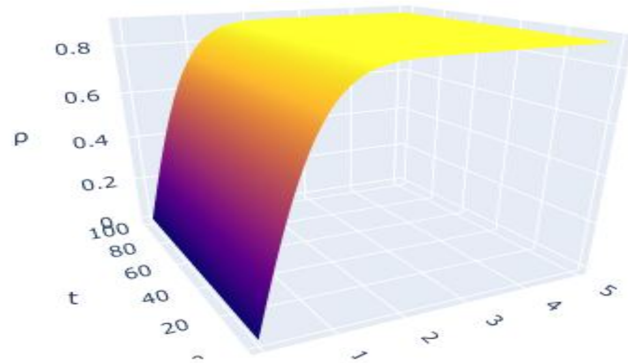
Source: Own elaboration.

Figure 8. Representation of $\vec{E}$ with the solution $e^{-x^2}((1 - 0.00004\ Cos(0.036\ -\ 1.007t))$.



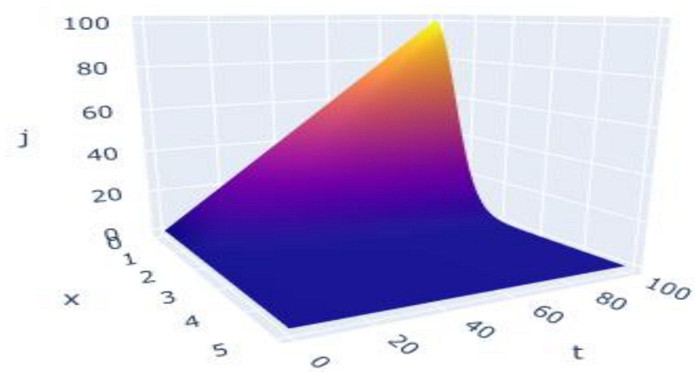
Source: Own elaboration.

Figure 9. Representation of ρ with $2e^{-x^2}((1 - 0.00004\, Cos(0.036\ -\ 1.007t))$.



Source: Own elaboration.

Figure 10. Representation of $\vec{J}$ with $-0.00004e^{-x^2}Sin(0.036\ -\ 1.007t)$.



Source: Own elaboration.

With the ansatz of the electric field $\vec{E}$, it is a straightforward matter to take the derivative of it with respect to x to obtain the charge distribution:

$$\rho = -2Ax\, \epsilon_0 e^{-x^2}(1 - B\ Cos(c - t))$$

which is shown in Figure 9. Additionally, the currency flow in the market can be directly obtained by taking the derivative of $\vec{E}$ with respect to t and multiplying by μ_0, which yields

$$J = -AB\ e^{-x^2}\ Sin(c - t)/\mu_0$$

and is shown in Figure 10.

7.1. Discussion

The results obtained in this study, based on an analogy between gauge theory and the behavior of the foreign exchange market, offer a novel perspective for modeling out-of-equilibrium dynamics in financial systems. This approach captures price fluctuations and identifies arbitrage opportunities as manifestations of curvature within a gauge space. The empirical analysis of the euro-dollar exchange rate during the first half of 2023 demonstrates that the observed gains can be effectively modeled using a gauged diffusion equation within the structure of gauge symmetry. This approach is further supported by nonlinear diffusion models, particularly stochastic differential equations, which have proven effective in capturing the complexity of price dynamics in foreign exchange markets

(Jäger & Kostina, 2006). In contrast with traditional time series models such as ARIMA or GARCH, which assume near-equilibrium conditions, our approach successfully represents the nonlinear and speculative nature of highly liquid markets like FOREX (Engel & West, 2005; Krugman, 2008). Recent studies further suggest that, unlike other currency pairs, the USD/EUR exchange rate does not exhibit clear mean-reverting behavior, reinforcing the need for more flexible modeling frameworks (Choi & Lee, 2020). Additionally, monetary policy decisions and macroeconomic conditions introduce complex dynamics that equilibrium-based models often overlook (Lian et al., 2011; Young, 1999). Nevertheless, while diffusion models offer a robust framework for representing such dynamics, their implementation may pose challenges in highly volatile environments due to difficulties in model calibration and interpretation (Jäger & Kostina, 2006).

Our model finds conceptual correspondence with previous proposals, such as (Young, 1999; Ilinski, 2000). Our work complements these perspectives by operationalizing the analogy with real data and fitting specific functions to the electric field, allowing dynamic visualization of profits and their physical-mathematical sources. The application of physics-based tools to finance has raised important concerns in literature. For instance, in Ref. (Sornette, 1998) it is questioned the empirical relevance of gauge theory in efficient markets, arguing that arbitrage opportunities arise more from uncertainty than from geometric curvature. This critique has been echoed in (Keen, 2002), that argued that gauge-based models may overlook the inherent complexity of agent behavior, and in (Rosser, 2006), that contended that econophysics often disregards the theoretical richness of traditional economics, potentially leading to oversimplified interpretations of market dynamics.

Nevertheless, more recent works, such as (Paolinelli & Arioli, 2018; ibid, 2019), have validated approaches derived from quantum and gauge physics to model price dynamics with greater precision than traditional methods, demonstrating that it is possible to successfully integrate physical concepts into financial analysis, representing complex pricing structures and nonlinear behaviors that conventional models fail to capture adequately (Wu, n.d.). Moreover, empirical studies have identified power-law distributions in financial markets, which contradict the assumptions of the efficient market hypothesis and support the relevance of physics-inspired modeling frameworks (Mantegna & Stanley, 2002). Despite these advancements, skepticism persists regarding the effectiveness of physics-based models to fully capture the subjective, institutional, and psychological dimensions of financial behavior. Therefore, more integrative approaches that combine the formal rigor of physics with the theoretical and behavioral insights of economics are required in this direction.

Other studies (Kholodnyi, 2002; Kholodnyi & Trading, 2003) introduced gauge symmetry models based on the beliefs and preferences of the agents in the market, opening new avenues for integrating subjective expectations within physically inspired frameworks. Moreover, in (Smolin, 2009) it was also proposed a “statistical economics” approach grounded in gauge principles, emphasizing out-of-equilibrium dynamics. Within the Mexican context, recent works (Ramos-Llanos & Raya, 2022; Servín-Tomás, et al., 2023), have demonstrated the applicability of simplified gauge models to the analysis of the local currency market, particularly in relation to triangular arbitrage. Our present study broadens this scope to include international currencies and incorporates a quantitative treatment of real exchange rate time series.

The inclusion of the damped cosine model to represent gains in terms of an electric field, facilitates the derivation of charge density and monetary flow. These quantities, from an economic perspective, allow speculative dynamics and price adjustments to be interpreted as processes analogous to interactions between charges and fields, reinforcing the validity of the proposed interdisciplinary framework (Maldacena, 2015). This approach aligns with the physics classical electrodynamics, modeling market behavior using electric fields, and has been supported by recent models that use quantized gauge fields to represent financial variables such as interest and exchange rates as geometric structures (Ilinski, 1997; De Zela, 2023).

On physical grounds, the electric field arises as the gradient of a potential, and its intensity indicates the force exerted on a charge. When this concept is translated to the financial context, the electric field represents the magnitude and direction of the economic incentive acting on agents participating in the market. In this model, arbitrage profits are interpreted as a "force" that drives monetary flows along closed currency loops. Thus, the electric field is not merely an analogy, but a variable that quantifies the profit opportunity as a function of local exchange rate imbalances. The damped cosine-shaped field, empirically fitted to the data, captures periodic oscillations in these opportunities, allowing for the modeling of how arbitrage conditions emerge and dissipate, based on geometric principles.

Finally, this methodology offers considerable advantages in terms of simulation and prediction. Unlike conventional statistical models, gauge theory allows the incorporation of external disturbances such as interest rate changes or geopolitical shocks as dynamic sources of curvature. This provides a flexible and powerful framework for analyzing the propagation of such effects through financial systems, as demonstrated in recent studies on fast monetary dynamics (Sokolov, Kieu, & Melatos, 2010) and high-frequency stochastic geometry (Dupoyet, Fiebig, & Musgrove, 2010). Furthermore, this framework is argued to outperform traditional econometric models in capturing high-volatility behaviors, as it more effectively reflects abrupt market reactions to exogenous events (Yang, 2015; Juárez, n.d.). However, it is important to acknowledge potential limitations of this approach: The mathematical complexity of gauge-based models can hinder their empirical implementation, and the direct translation of physical concepts into economic contexts may not always capture the institutional, behavioral, or regulatory nuances of financial systems. The discussion of our results confirms that gauge theory-inspired models represent a promising tool for understanding, simulating, and anticipating complex behaviors in the currency exchange market. Although idealized, this approach makes a significant contribution to the nonlinear and dynamic modeling of prices, complementing classical econometric methods with a more geometric, structural, and realistic perspective

8. Conclusions

This study proposes a reinterpretation of the foreign exchange market through the lens of physics, specifically by considering gauge theory principles to model currency arbitrage in the FOREX market. The central motivation of the manuscript is to provide an alternative theoretical and mathematical tool to describe out-of-equilibrium market behaviors that traditional econometric models often fail to capture.

By analyzing actual EUR/USD exchange rate data from January to June 2023, we demonstrated that the analogy with a one-dimensional electrodynamic system enables a consistent representation of gains as electric fields and connects them with "economic charge" densities. This interpretative framework aligns with earlier works (Ilinski, 2001; Young 1999; Maldacena, 2015), and others, which described exchange rates as gauge connections and arbitrage as manifestations of curvature in a gauge space.

The results obtained from the modeling—specifically the damped cosine-based fitting of gain profiles, the derivation of charge density ρ, and the computation of monetary flow—reinforce the conceptual validity of the gauge-based approach. They also corroborate the proposal of (Vázquez and Farinelli, 2009), which defined arbitrage opportunities as non-zero curvature in a geometric space. While the model proposed in this manuscript is idealized, it effectively captures features such as transient imbalances, speculative profit cycles, and non-equilibrium behavior observed in real markets. Nevertheless, the research has several limitations. First, the model is constrained to a one-dimensional framework, omitting the complexity and topology of multi-currency networks. Second, it uses a quenched approximation, assuming static market rules that do not evolve over time. Third, it does not include microstructural elements such as transaction costs, spreads, or institutional frictions. Lastly, the model has not been empirically validated through calibration or statistical backtesting against competing models.

Despite these constraints, the study offers a meaningful conceptual contribution. It demonstrates that financial variables such as exchange rates, profits, and arbitrage flows can be mapped onto physically meaningful structures, opening a pathway for the application of gauge theory in quantitative finance. The presence of free parameters in the equations allows adaptation to market-specific conditions, potentially increasing descriptive and predictive accuracy.

The justification for this work lies in the need to develop models that more accurately reflect the volatility, complexity, and inherent disequilibrium of modern financial systems. Gauge theory, with its well-established mathematical structure, provides a rich foundation for modeling speculative behavior, systemic feedback, and the propagation of shocks across interconnected markets. Future research will address several directions: (1) implementing numerical solutions of the gauged diffusion equation for real-time prediction of exchange rate movements; (2) extending the model to include multiple currencies and arbitrage loops of greater complexity; (3) incorporating exogenous shocks as dynamic sources of curvature; and (4) empirically estimating the propagation constant $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

as a scale for measuring the diffusion speed of financial disturbances across the network.
This study opens a promising line of inquiry within the field of econophysics applied to international finance, showing that theoretical tools from physics can enrich our understanding of market structure and dynamics, and lay the groundwork for innovative approaches to financial modeling.

References

[1] Arthur, W. B. (2021). Foundations of complexity economics. Nature Reviews Physics, 3(2), 136-145. https://doi.org/10.1038/s42254-020-00273-3

[2] Baaquie, B. E. (2007). Quantum finance: Path integrals and Hamiltonians for options and

interest rates. Cambridge University Press. https://doi.org/10.1017/CBO9780511617577

[3] Canton, H. (2021). Bank for international settlements—BIS. In The Europa Directory of International Organizations 2021 (pp. 468-470). Routledge.

[4] Chaboud, A., Rime, D., & Sushko, V. (2023). The foreign exchange market. In Research Handbook of Financial Markets (pp. 253-275). Edward Elgar Publishing. https://doi.org/10.4337/9781800375321.00020

[5] Choi, S., & Lee, J. (2020). Maximum Likelihood Estimation of Continuous-time Diffusion Models for Exchange Rates. East Asian Economic Review, 24(1), 61–87. https://doi.org/10.11644/KIEP.EAER.2020.24.1.372

[6] De Zela, F. (2023). Neoclassical models of charged particles. Journal of Physics Communications, 7. https://doi.org/10.1088/2399-6528/acefa6

[7] Dupoyet, B., Fiebig, H. R., & Musgrove, D. P. (2010). Gauge invariant lattice quantum field theory: Implications for statistical properties of high frequency financial markets. Physica A: Statistical Mechanics and its Applications, 389(1), 107-116. doi:10.1016/j.physa.2009.09.002

[8] Engel, C., & West, K. D. (2005). Exchange rates and fundamentals. Journal of political Economy, 113(3), 485-517. doi:10.1086/429137

[9] Jackson, M. O. (2009). Networks and economic behavior. Annu. Rev. Econ., 1(1), 489-511. doi:10.1146/annurev.economics.050708.143238

[10] Jäger, S., & Kostina, E. (2006). An Inverse Problem for a Nonlinear Stochastic Differential Equation Modeling Price Dynamics. https://doi.org/10.11588/HEIDOK.00006549

[11] Juárez, G.S. (n.d.). Gauge Theory on Financial Modeling. https://doi.org/10.24275/uam/azc/dcsh/efr/2015v5n2/sierra

[12] Fant, L. F. (1999). Investment behavior of mutual fund shareholders: The evidence from aggregate fund flows. Journal of Financial Markets, 2(4), 391-402. https://doi.org/10.1016/S1386-4181(99)00006-3

[13] Farinelli, S. (2015). Geometric arbitrage theory and market dynamics. Available at SSRN 1113292. doi:10.3934/jgm.2015.7.431

[14] Georgescu-Roegen, N. (1971). The entropy law and the economic process. Harvard university press.

[15] Harris, L. (2002). Trading and exchanges: Market microstructure for practitioners. Oxford university press. https://doi.org/10.1093/oso/9780195144703.003.0013

[16] Ilinski, K. (1997). Physics of Finance. arXiv: High Energy Physics - Theory. https://arxiv.org/abs/hep-th/9710148 doi:10.48550/arXiv.hep-th/9710148

[17] Ilinski, K. N. (1998). Gauge Physics of Finance: simple introduction. arXiv preprint cond-mat/9811197. doi:10.48550/arXiv.cond-mat/9811197

[18] Ilinski, K. (2000). Gauge geometry of financial markets. Journal of Physics A: Mathematical and General, 33(1), L5. DOI 10.1088/0305-4470/33/1/102

[19] Ilinski, K. (2001). Physics of finance: Gauge modelling in non-equilibrium pricing. John Wiley & Sons.

[20] Kholodnyi, V. A. (2002). Valuation and dynamic replication of contingent claims in the framework of the beliefs-preferences gauge symmetry. The European Physical Journal B-Condensed Matter and Complex Systems, 27(2), 229-238. DOI: 10.1140/epjb/e20020148

[21] Kholodnyi, V. A., & Trading, T. E. (2003). Beliefs-Preferences Gauge Symmetry and Dynamic Replication of Contingent Claims in a General Market Environment. Dynamics of Continuous,

Discrete & Impulsive Systems Series B, 10, 81–98.

[22] Keen, S. (2002). Debunking efficient markets. Quantitative Finance, 2(6), 406–407. https://doi.org/10.1080/14697688.2002.0000005

[23] Krugman, P. R. (2008). International economics: Theory and policy, 8/E. Pearson Education India.

[24] Lian, D., Chao-liang, H., & De-hai, Z. (2011). Empirical analysis on the dynamic system of dollar-euro exchange rate — A redesigned method based on the nonlinear dynamic systems. International Conference on Management Science and Engineering, 882–889. https://doi.org/10.1109/ICMSE.2011.6070064

[25] López-Chacón, J. Raya, A., & Flores-Romero, M. B. Teoría de Norma en el Mercado Financiero para el Arbitraje de Divisas en el Mercado Forex, Tesis (2022), Available at: https://www.inidem.edu.mx/assets/tesis_jennifer-lopez-chacon-teoria-de-norma-en-el-mercado-financiero-para-el-arbitraje-de-divisas-en-el-mercado-forex.pdf

[26] Mantegna, R.N., & Stanley, H. E. (2002). Investigations of Financial Markets Using Statistical Physics Methods (pp. 352–371). Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-642-56257-0_12

[27] Malaney, P. N. (1997). The index number problem: A differential geometric approach. Harvard University.

[28] Maldacena, J. (2015). The symmetry and simplicity of the laws of physics and the Higgs boson. European Journal of Physics, 37(1), 015802. https://doi.org/10.1088/0143-0807/37/1/015802

[29] Mishkin, F. S., & Eakins, S. G. (2006). Financial markets and institutions. Pearson Education India.

[30] Morisawa, Y. (2009). Toward a Geometric Formulation of Triangular Arbitrage: —An Introduction to Gauge Theory of Arbitrage—. Progress of Theoretical Physics Supplement, 179, 209-215. https://doi.org/10.1143/PTPS.179.209

[31] Paolinelli, G., & Arioli, G. (2018). A path integral based model for stocks and order dynamics. Physica A: Statistical Mechanics and its Applications, 510, 387-399. DOI: 10.1016/j.physa.2018.07.007

[32] Paolinelli, G., & Arioli, G. (2019). A model for stocks dynamics based on a non-Gaussian path integral. Physica A: Statistical Mechanics and its Applications, 517, 499-514. DOI: 10.1016/j.physa.2018.11.044

[33] Peskin, M. E. (2018). An Introduction to quantum field theory. CRC press. doi:10.1201/9780429503559

[34] Ramos-Llanos L. A. & Raya, A. (2022). Oportunidades de arbitraje desde la perspectiva de teorías de norma, Tesis(2022), https://bibliotecavirtual.dgb.umich.mx:8083/xmlui/handle/DGBUMICH/12210.32.

[35] Raya, A., Romero, M. F., & Chacón, J. L. (2025). Teorías de gauge (norma) en los mercados financieros, una revisión histórica. Revista Mexicana de Física E, 22(1 Jan-Jun), 010302-1. https://doi.org/10.31349/RevMexFis.22.010302

[36] Rodrigues, N. D. (2019). Application of gauge theory to finance: a systematic literature review. PQDT-Global.

[37] Rochimah, A., & Yuliana, I. (2025). Capital Market and Macroeconomic Reaction to The Hamas-Israel Conflict 2023. Revista mexicana de economía y finanzas, 20(4). https://doi.org/10.21919/remef.v20i4.1010

[38] Rosser, J. B. (2006). Dynamics of markets: Econophysics and finance. Discrete Dynamics in

Nature and Society, 1–7. https://doi.org/10.1155/DDNS/2006/97682

[39] Schwichtenberg, J. (2019). Physics from Finance: A gentle introduction to gauge theories, fundamental interactions and fiber bundles. No-nonsense books.

[40] Servín-Tomás, C. J., Raya, A., & López-Chacón, J. (2023). Física y finanzas [Tesis de maestría, Universidad Michoacana]. https://bibliotecavirtual.dgb.umich.mx:8083/xmlui/handle/DGBUMICH/16781.34

[41] Shalizi, C. (2001). Physics of Finance: Gauge Modelling in Non-Equilibrium Pricing. Quantitative Finance, 1(4), 393.

[42] Smith, A. (1776). An inquiry into the nature and causes of the wealth of nations: Volume One. London: printed for W. Strahan; and T. Cadell, 1776..

[43] Smolin, L. (2009). Time and symmetry in models of economic markets. arXiv preprint arXiv:0902.4274. doi:10.48550/arXiv.0902.4274

[44] Sokolov, A., Kieu, T., & Melatos, A. (2010). A note on the theory of fast money flow dynamics. The European Physical Journal B, 76(4), 637-642. https://doi.org/10.1140/epjb/e2010-00223-2

[45] Sornette, D. (1998). Gauge theory of Finance?. International Journal of Modern Physics C, 9(03), 505-508. https://doi.org/10.1142/S0129183198000406

[46] Stillwell, J., & Stillwell, J. (1989). Mathematics and its history (Vol. 3). New York: Springer. https://doi.org/10.1007/978-1-4684-9281-1

[47] Yang, Y. (2015). Gauge field theory of market dynamics: Toward a solution of the “man vs. men” dilemma. https://mpra.ub.uni-muenchen.de/65015/

[48] Young, K. (1999). Foreign exchange market as a lattice gauge theory. American Journal of Physics, 67(10), 862-868. https://doi.org/10.1119/1.19139

[49] Vairasigamani, P., & Amilan, S. (2025). Unveiling the dynamic linkages and hedging between Indian sectoral indices and alternative investments in crisis episodes. Revista mexicana de economía y finanzas, 20(3). https://doi.org/10.21919/remef.v20i3.1211

[50] Vázquez, S. E., & Farinelli, S. (2009). Gauge invariance, geometry and arbitrage. arXiv preprint arXiv:0908.3043. doi:10.48550/arXiv.0908.3043

[51] Von Neumann, J., & Morgenstern, O. (2007). Theory of games and economic behavior: 60th anniversary commemorative edition. In Theory of games and economic behavior. Princeton university press.

[52] WU, H. (n.d.). The Influence of Physics on Finance. https://doi.org/10.3969/j.issn.1006-1096.2007.06.038

[53] Zhou, S., & Xiao, L. (2010). An application of symmetry approach to finance: Gauge symmetry in finance. Symmetry, 2(4), 1763-1775. https://doi.org/10.3390/sym2041763